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OPEN Contrasting dynamics of past climate states and critical transitions via dimensional analysis

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While past major climate transitions can be unequivocally identified, understanding of underlying mechanisms and timescales remains limited. We employ a dimensional analysis of benthic stable isotope records across different timescales to uncover how Cenozoic climatic fluctuations are associated with changes in the number of feedbacks and mechanisms involved. Our analysis indicates that warmer and colder climates respond substantially differently to orbital forcing. Notably, large numbers of feedbacks dominated during the Icehouse (3.3 Ma to present) state at obliquity and eccentricity timescales, and during the Warmhouse (66–56 Ma and 47–34 Ma) and Hothouse (56–47 Ma) states at precession timescales. During the Coolhouse (34-3.3 Ma) state the number of active feedbacks was low and had no dominant timescale. Coupling between climate signals that affect oxygen and carbon isotope records appears high only in the Icehouse state, and low to absent in all other states. We also find that anomalously high active feedback numbers and very high coupling occurred across all timescales during the Paleocene-Eocene Thermal Maximum (PETM, 56 Ma), which suggests a complete system perturbation. In conclusion, our findings challenge the notion of a simple and unique conceptual model of interconnected feedbacks in reproducing Cenozoic paleoclimate variability, given that different numbers of active feedbacks with different levels of coupling governed different timescales between climate states, which then affected the inherent (in-)stability of each climate state.

Keywords Cenozoic climate variability, Abrupt climate shifts, Dimensional analysis

Earth's climatic history has been reconstructed using sediment archives from both marine and terrestrial environments. In particular, since the 1970s the development of high-resolution deep-sea oxygen (δ^{18} O) and carbon (δ^{13} C) isotope records has significantly enhanced the understanding of past climate trends, cyclic variations, rates of change, and transient events throughout the Cenozoic era (66 My ago to present)^{1,2}. However, these compilations have partially fallen short in accurately documenting the full range and detailed characteristics of Cenozoic climate variability, due to gaps and insufficient dating accuracy and temporal resolution, especially for the period before 34 My ago. A recent study³ addressed these challenges by taking advantage of sediment archives obtained by the International Ocean Discovery Program (IODP) and its predecessor programs (DSDP, ODP) to compile and analyze a comprehensive, new composite record of carbon and oxygen isotopes in deep-sea benthic foraminifera, which was precisely tuned to astronomical cycles. The resulting climate reference curve, CENOGRID (CENOzoic Global Reference benthic foraminiferal carbon and oxygen Isotope Dataset)³, provides high-resolution coverage of the past 66 My, enabling the detection of long-term Cenozoic climate variability (Figs. 2, 3A).

Employing statistical recurrence analysis (RA) on the CENOGRID record, Westerhold et al.³ identified four key climatic regimes in the Cenozoic era: Hothouse, Warmhouse, Coolhouse, and Icehouse, each demonstrating

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unique and statistically distinct features³. The Warmhouse and Hothouse phases, spanning from the Early Paleocene (~66 million years ago) to the Late Eocene (~34 million years ago), were marked by extremely high atmospheric CO_2 concentrations, leading to global mean temperatures significantly higher than today^{4–8}. The Earth was largely ice-free, with polar regions devoid of ice sheets and ocean circulation likely sluggish due to reduced thermal gradients. This regime also included extreme warming events, such as the Paleocene-Eocene Thermal Maximum (PETM) around 56 million years ago, which was associated with rapid climate shifts and ocean acidification. However, the late phase of the Warmhouse state that started around 40 million years ago exhibited a gradual cooling trend. The first signs of Antarctic glaciation emerged, marking a transition toward cooler conditions. This phase culminated in the Eocene-Oligocene Transition (EOT), a pivotal event around 34 million years ago that marked the permanent glaciation of Antarctica and a fundamental shift in Earth's climate system⁸⁻¹². The Coolhouse regime, spanning from the Oligocene (\sim 34 million years ago) to the Pleistocene (\sim 3.3 million years ago), saw further decline in CO_2 , with decreases in global temperature, and a significant expansion of the Antarctic ice sheets, while early ice formation began in the Northern Hemisphere^{13,14}. This period saw major climatic reorganizations, particularly during the Mid-Miocene Climate Transition (MMCT) around 14 million years ago, when a further cooling trend and enhanced glaciation set the stage for modern ice age dynamics¹⁵. Finally, the Icehouse regime, which extends to the present, is dominated by alternating glacial and interglacial cycles, driven primarily by orbital variations described by Milankovitch theory. Large ice sheets persist in both hemispheres, particularly in Antarctica and Greenland, and ocean circulation plays a crucial role in modulating climate variability on both short and long timescales¹⁶. The Pleistocene glacial cycles, which began around 3.3 million years ago, define this most recent phase of Earth's climate history, illustrating the strong coupling between orbital forcing, ice sheet dynamics, and atmospheric greenhouse gas concentrations.

In addition to the critical transitions between the four macroclusters of climate variability mentioned above, the climate of the Cenozoic experienced several other transitions associated with tipping points¹⁷ of the Earth system. In Rousseau et al.¹⁸ such transitions were characterized by combining recurrence analysis of the individual time series^{19,20} with a multivariate analysis based on the quasi-potential theory^{21,22}. This analysis showed that tipping behaviour can emerge in a large variety of fashions and on rather different time scales. Additionally, it showed that the use of more than one proxy data is key to discover the full range of critical transitions. Indeed, through the analysis of the benthic δ^{18} O nine major climatic tipping points ($TP_O 1-TP_O$ 9) are identified (continuous white lines in Figs. 2, 3). Two significant warming events occurred at \sim 58 Ma (TP_{O2}) and ~56 Ma (TP_{O3}) , associated with the late Paleocene-Eocene hyperthermal and the Paleocene-Eocene Thermal Maximum (PETM). This was followed by progressive cooling, with key transitions at \sim 47 Ma $(TP_O 4, Early-Middle Eocene cooling)$, ~40 Ma $(TP_O 5, marking sustained CO_2 decline)$, and ~34 Ma $(TP_O 6, P_O 6)$ Eocene-Oligocene Transition, linked to Antarctic ice sheet formation). After TP_O6 , the Earth entered a colder climate mode, with further transitions at \sim 14 Ma (TP₀7, Middle Miocene Climate Transition) and \sim 2.8 Ma (TP_Q) , onset of Northern Hemisphere glaciations). These events correspond to changes in global mean sea level (GMSL), carbonate compensation depth (CCD), and atmospheric CO₂ levels. Parallel analysis of the δ^{13} C record identifies 10 additional tipping points (TP_C1–TP_C10), capturing changes in ocean carbon cycling. Notably, the PETM is reflected in both δ^{18} O and δ^{13} C records (TP_C3 and TP_C3), as is the Eocene-Oligocene Transition (TP_O6 and TP_C8). Some tipping points appear to be crossed multiple times in a dynamic fashion, suggesting complex feedbacks in the climate system.

Here we aim to further advance the investigation of the Cenozoic climate by applying a multiscale and bivariate dimensional analysis of the CENOGRID record^{23,24}. Our goal is to characterize the four main macroclusters of climate variability in terms their feedbacks, associated timescales, and stability/predictability. We will devote special attention to the study of the critical transitions highlighted in Rousseau et al.¹⁸ to extracting information on the relationship between the δ^{13} C and δ^{18} O signals across a variety of timescales; see a schematic in Fig. 1).

Although there is an extensive literature on the role of astronomical forcing to climate dynamics and on classifying the different climate states in terms of their stability (e.g., Ref.^{3,18}), a detailed analysis of the local (in time) and timescale-dependent properties of stability of the climate dynamics is still lacking. Here we aim to study the variability at different timescales within each macrocluster of climate variability defined in Westerhold et al.³, following Rousseau et al.¹⁸ where it was suggested that the proxy record has distinct multiscale features.

The type of analysis we perform here has shown remarkable effectiveness in various applications^{23,24}. In our case, through the Empirical Mode Decomposition (EMD) we decompose CENOGRID δ^{13} C and δ^{18} O records into intrinsic oscillating functions at different average timescales²⁵ and then we determine, across timescales (τ), three key dynamical parameters from Extreme Value Theory (EVT)²⁶⁻²⁸:

- 1. Instantaneous dimension *d*: positive anomalies with respect to a reference value indicate a larger prevalence of positive and/or reduced presence of negative feedbacks;
- 2. Extremal index $0 \le \theta \le 1$, where low θ values indicate that the system is highly persistent about a reference state, whereas larger θ values indicate low persistence;
- 3. Co-recurrence ratio α , which quantifies the mutual coupling between two proxies, with α close to 1 meaning stronger coupling, while α close to 0 indicates weaker coupling.

It is well known that, depending on the timescale of interest, the same system might exhibit different stability properties, depending on whether positive or negative feedbacks dominate²⁹.

We remark that the information we draw from looking at the instantaneous dimension should be interpreted in relative rather than absolute terms. Rather than taking the obtained estimate of d at face value, we proceed as follows. If, e.g., state (a) features a larger value for d than state (b), then we conclude that the balance between positive and negative feedback mechanisms of state (a) is more tilted towards instability.



Fig. 1. Schematic of the key features of the four main climate states. The dominant characteristics of the four climate macro-states as a function of their characteristic isotopic perturbation ranges in ‰ compared to present (with different colors and the indication of their time extension) in terms of number and timescales of effective feedbacks, stability/predictability, and δ^{13} C- δ^{18} O coupling at multi-millennial timescales.

Furthermore, if, e.g., state (c) features a larger value of d at timescale τ_1 than at timescale τ_2 , then we rank τ_1 as being associated with the primary/dominant mode of variability. We also emphasize that lower θ values do not mean that the system is not oscillating at that timescale but that it can (stably) oscillate at that timescale²⁷.

These metrics, while seemingly abstract, serve a valuable role in revealing the underlying physical processes within a system, reflecting the capability of the climate system to respond to external forcings. In climate dynamics, such insights are essential for understanding phenomena like abrupt climate transitions or the development of distinct climate patterns, which result from the interplay of multiple processes operating on different timescales. Finally, both EMD and EVT analyses are resilient to non-uniform data spacing and resolution variability, reinforcing that observed stability properties are intrinsic to climate dynamics rather than artifacts of data processing. Key aspects of the EMD, an in-depth description of the uni-variate and bi-variate parameters, and our scale-dependent procedure are reported in the Methods.

Multiscale analysis of the CENOGRID dataset

The behavior of our multiscale bivariate metrics (Figs. 2, 3) highlights key properties of the climate variability recorded in the CENOGRID dataset.

The Coolhouse is generally characterized by lower values of the instantaneous dimension d (Figs. 2, 3B) than the other climate states, except for an increase during specific events such as the Monterey positive carbon isotope excursion between 16.9 and 13.5 My ago³⁰ that lacks dominant expression of any specific orbital period. The timescale-dependent estimate of d reveals that, while the Warmhouse and the Hothouse states are dominated by active positive feedbacks (larger d) at short orbital timescales (mainly precession), the Icehouse is dominated by active positive feedbacks at obliquity and eccentricity timescales. Furthermore, we also depict a non-negligible role of sub-orbital (i.e., shorter than precession) processes in both warm climates as well as during the Icehouse, which is lacking in the Coolhouse. For warm climates this can be due to moderate-tohigh carbon dioxide levels and the existence of a mix of tropical and temperate ecosystems observed during both the Hothouse and the Warmhouse. Beyond astronomical forcings as specific drivers of the climate system, additional potential mechanisms may operate at shorter timescales with respect to the precession cycle relate to a combination of volcanic activity, greenhouse gas emissions, and feedback mechanisms in the carbon cycle^{31,32}. These features are also evident from the uni-variate analysis (Supplementary Figures S1, S2) which shows larger d for the δ^{13} C than δ^{18} O. Conversely, for the Icehouse this can be related to the presence of enhanced polar ice sheets and glaciers, enhancing ice-ocean-atmosphere coupling processes, and associated greenhouse gas variability, leading to short-term variability. Furthermore, before and during the Monterey Excursion (16.9-13.5 Ma)³³ within the Coolhouse the enhanced positive feedbacks at sub-orbital timescales can be related to major climatic, sea-level, and ecological shifts driven by temperature variations, with the global climate oscillating between cooler and warmer phases. Indeed, modeling suggests that low- and mid-latitude processes in the climate system respond in a nonlinear way to insolation forcing^{34,35}, with the hydrological cycle and its highly seasonal precipitation patterns acting as positive feedback during intervals of strong monsoon response to shortterm insolation change, which could play a major role in the global distribution of moisture and energy³⁶. These features are again evident from the uni-variate analysis (Supplementary Figures S1, S2) which shows larger d for δ^{18} O than for δ^{13} C. Moreover, the general trend of increasing *d* with decreasing timescale during the Icehouse can be associated with initially relatively small ice volume changes, which contributed to less extreme climate







Fig. 3. Multiscale bivariate analysis during the colder periods (last 34 Ma). (**A**) Time series of the CENOGRID paleoclimate records of $\delta^{13}C$ (blue) and $\delta^{18}O$ (red). Bi-variate multi-scale metrics: (**B**) instantaneous dimension *d*, (**C**) extremal index θ , and (**D**) co-recurrence index α . Vertical dashed lines in panel (**A**) mark the different geological epochs, while specific events are reported when they occurred. Vertical continuous and dotted lines in panels (**B**)–(**D**) indicate the Tipping Points (TPs) identified by Rousseau et al.¹⁸ with a uni-variate approach for $\delta^{18}O$ and $\delta^{13}C$, respectively (for visual purposes we only label TPs for $\delta^{13}C$). The horizontal dashed-dotted white lines refer to Milankovitch timescales of precession (~ 23 ky), obliquity (~ 41 ky), and eccentricity (~ 100 ky), respectively. We note that θ close to 0 (1) means a more (less) stable/ persistent state, while α close to 0 (1) means a less (more) mutual coupling.

(2025) 15:13224

Scientific Reports |

fluctuations than they do later, when the dominance shift from 40 ky cycles to 100 ky cycles occurred. In the regular alternation between long glacial periods and shorter, warmer interglacial periods after the MPT, larger values of d may be linked with more extreme temperature fluctuations³⁷.

In general, we observe substantial mutual coupling (increased α ; Figs.2, 3D) on relatively short time scales ($\tau \lesssim 23$ ky) between δ^{13} C and δ^{18} O during warm climate states, especially after the PETM and during specific intervals such as the Late Danian, Early Eocene Hyperthermals, which no longer occurs during cold climate states, apart from temporarily intensified coupling over the middle Miocene Monterey event. This suggests that sub-orbital variability was mainly driven by processes involving positive temperature feedbacks, such as icealbedo interactions (which can be parameterized as a function of the temperature, e.g., Ref.^{38,39}) or greenhouse variations due to volcanic activity³¹. Volcanic activity modulates atmospheric greenhouse gas concentrations through episodic CO₂ emissions and aerosol-driven radiative forcing. In this scenario, volcanic perturbations act as external drivers that can trigger internal feedback within the carbon-climate system. For instance, increased CO₂ due to sustained volcanic degassing can enhance weathering processes, alter ocean carbon storage, and modify biological productivity, all of which are further mediated by physical and biogeochemical feedbacks.

During the Icehouse, starting from the M2 glaciation at around 3.2 Ma, prolonged mutual coupling is instead found at obliquity and eccentricity timescales, which did not persistently occur before. This is related to the repeated glacial-interglacial variations, with strong coupling between mean surface temperature and carbon cycle intensity, suggesting that α can be used as a key parameter to identify intervals when the system is strongly forced. Furthermore, while all climate states are characterized by low-stability behavior (high θ ; Figs. 2, 3C) at timescales shorter than the obliquity period, increased stability/persistence (lower θ) is observed at obliquity and eccentricity timescales

Finally, our analysis highlights the exceptional nature, even among other hyperthermals, of the Paleocene Eocene Thermal Maximum (PETM), where *d* is high and almost constant across all timescales and lacks association with any particular orbital timescale, while coupling is high at all timescales (high α ; Fig. 2D). This can be related to the origin of the PETM which was characterized by sudden release of methane hydrates from ocean sediments and intense volcanic CO₂ release, leading to global warming. The onset of the PETM was rapid, occurring within a few thousand years, while its decline was more gradual⁴⁰ (Fig. 2A). In contrast, the Eocene-Oligocene Transition (EOT, ~34 Ma) seems to be primarily influenced by processes at precession timescales, marking the end of the precession-dominated Hot-/Warmhouse period (Fig. 3B–D). Both these transitions were identified as key abrupt transitions (Tipping Points, TPs) associated with major regime shifts that separate clusters of climate variability¹⁸, and here we add insight about active feedbacks and associated reference timescales.

Dynamical features of the four climate variability macroclusters across timescales

To facilitate the interpretation of our results, we investigate - see Fig. 4 - the average values of d, θ , and α in the timescale domain (τ) over each of the four macroclusters of variability shown in Fig. 1.

The Hothouse climate state is characterized by d decreasing with τ , except for a plateau around the 100-ky eccentricity period. This indicates a dominance of positive feedbacks in the climate response at around the 23-ky precession timescale. Thus, the primary mode of variability of the Hothouse climate is recognized to be associated with precession. A similarly decreasing trend with τ is visible for θ , which suggests increased climate stability at the primary mode of variability (precession). Finally, α is anomalously large at precession timescales, as well as at timescales larger than the eccentricity period. While the former cannot be attributed to a specific mechanism, the latter mainly is a signature of the PETM.

The Warmhouse state is also characterized by decreasing d and θ with increasing τ . Additionally, d values are in general lower and θ values are in general comparable with respect to the Hothouse. As opposed to the Hothouse, the Warmhouse is characterized by approximately constant $\alpha \approx 0.1$ for all timescales, which indicates low mutual coupling between δ^{13} C- δ^{18} O. Based on the d behavior, we identify precession as being linked to the primary mode of variability.

During the Coolhouse state there is no clearly dominant timescale for *d*. Instead, θ and α are comparable to those in the warm states. Such a decrease in θ and α with τ suggests more relatively stable conditions during the Coolhouse state. Based on the lack of timescale-dependence of *d* we are not able to find a primary mode of variability for the Coolhouse, except for an emerging importance of sub-orbital variability.

Finally, results are remarkably different for the ongoing Icehouse state, confirming the observed and detailed differences highlighted in Figs. 2, 3. During the present climate state, the values of the instantaneous dimension d are larger than other climate states over almost all the considered timescales, peaking at around $\tau \approx 41$ ky, indicating the relevance of the obliquity timescale in the present climate state. A plateau is also found around the 100-ky eccentricity timescale. This suggests more positive feedback mechanisms involved in climate responses at both 41- and 100-ky timescales, relative to the prior warm climate states and the Coolhouse state (centered on the 23-ky precession timescale). Thus, we indicate the obliquity and the eccentricity cycles as the primary and secondary modes of variability, respectively. Meanwhile, extremal index θ , while still characterized by a decrease with τ , indicates considerably greater stability than in the previous climate states. This is a reflection of the presence of stable climates likely related to stability of both the Antarctic and Greenland ice sheets. Additionally, α is highly timescale dependent, but now peaks at 41 and 100 ky timescales, reaching higher values than previously observed. This behavior may reflect an increase in ice-ocean-atmosphere interactions with the pacing of ice ages alternating large ice sheets highly varying through time¹².

Overall, our analysis implies that warmer and colder climates respond in substantially different ways to orbital forcing. Responses during warm climates are dominated by precession timescale variations, whereas cold climates appear to be characterized by responses on obliquity and eccentricity timescales, which highlights the unique features and nature of the ongoing Icehouse state.



Fig. 4. Time-averaged multi-scale bi-variate statistics during the four climate states identified by³. Temporal averages of the bi-variate multi-scale metrics: (**A**) instantaneous dimension $\langle d \rangle$, (**B**) extremal index θ , and (**C**) co-recurrence ratio α . The error bars refers to the inter-quartile range. The vertical dashed-dotted black lines refer to Milankovitch scales of precession (~ 23 ky), obliquity (~ 41 ky), and eccentricity (~ 100 ky), respectively. Colors refer to the different climate states, while the gray shaded bars refer to the full record. We note that θ close to 0 (1) means a more (less) stable/persistent state, while α close to 0 (1) means a less (more) mutual coupling.

To further inspect climate responses at orbital timescales, we investigate the 2-D (d, θ) parameter-space behavior and the probability distribution functions (pdfs) in the four climate states (Fig. 5), as well as in different geological epochs (Supplementary Figure S3) and in a uni-variate framework between two consecutive TPs identified by Rousseau et al.¹⁸ (Supplementary Figures S4, S5).

Moving across orbital timescales (Fig. 5A,D,G) there is a transition toward a different portion of the 2-D parameter-space between warm (red and orange dots) and cold (cyan and purple dots) climates, which is particularly clear for the Icehouse (see purple dots in Fig. 5A). Furthermore, for the Icehouse a wider spread in the range of values of both *d* (between 4 and 12) and θ (between 0.2 and 0.6) is observed at the eccentricity timescale, while a narrower θ -range occurs at the precession scale (between 0.5 and 0.7). A larger *d* range is seen during the part of the Coolhouse and the entire Icehouse; conversely, a more limited region in the *d*- θ space (with ranging *d* between 4-10 and θ within 0.65-0.85 and 0.55-0.75) is associated with both Hothouse and Warmhouse states, respectively This is confirmed by the *d* and θ pdfs for the four climate states (Fig. 5B,C,E,F,H,I).

The special nature of the Icehouse state is made clear by the *d* distribution across orbital timescales which peaks at ~ 10 for all Milankovitch cycles. The *d* distribution is also similar at all orbital timescales during the Coolhouse, albeit with a peak at lower values (~ 5). Conversely, the two warm states are characterized by different



Fig. 5. Multiscale bivariate scatter-plots of the metrics at Milankovitch scales. $d - \theta$ scatter plots colored by time instants at the three Milankovitch scales of precession (**A**), obliquity (**D**), and eccentricity (**G**), respectively. The distribution of the instantaneous dimension *d* over the four different climate states identified by³ at the three Milankovitch scales of precession (**B**), obliquity (**E**), and eccentricity (**H**), respectively. The distribution of the extremal index θ over the four different climate states identified by³ at the three Milankovitch scales of precession (**B**), and eccentricity (**I**), respectively. The distribution of the extremal index θ over the four different climate states identified by³ at the three Milankovitch scales of precession (**C**), obliquity (**F**), and eccentricity (**I**), respectively. We remark that θ close to 0 (1) means more (less) stable/persistent state.

d distributions across orbital timescales: a wider *d* spread occurs at the precession timescale, and more peaked distributions at obliquity and eccentricity timescales, which also shift toward lower values (\sim 5).

The θ pdfs confirm the uniqueness of the Icehouse state: for all orbital timescales, θ is lower (more stable) during the Icehouse and the shape of the pdf is completely different relative to those of the other states, which turn are very similar to each other.

Further analyses that highlight the unique nature of climate during the last \sim 3.3 My are presented in Supplementary Figures S3–S5. Our results confirm the interpretation of the time-evolution of Earth's Cenozoic climate system as a trajectory within a dynamic landscape that is characterized by multiscale features that delineate a hierarchy of metastable states and corresponding transitions¹⁸.

Conclusions and outlook

The Cenozoic era, spanning the last 66 million years, has witnessed significant changes in Earth's climate³, which materialized into a combination of smooth variations combined with a prevalence of a diverse array of critical transitions¹⁸. Understanding the physical processes involved in such variability is crucial for interpreting paleoclimate data and projecting future climate scenarios⁴¹. Our analysis clearly highlights the crucial impact of polar ice sheet formation and evolution in regulating global climate, and feedback mechanisms have been critical to shaping these ice sheets. For example, polar ice sheet growth causes enhanced reflection of sunlight back into space, which causes further cooling that, in turn, fosters further ice growth (the positive ice-albedo feedback). Moreover, ice sheet waxing and waning is crucial in ocean-atmosphere coupling, which is at the basis of heat transport across the globe and thermal regulation of climate. And ice sheet fluctuations also affected vegetation-zone displacements, which further affect surface albedo and, thus, the energy balance of climate^{38,39}.

We show here that the critical transitions identified in Ref.¹⁸ are accompanied by stronger positive feedback mechanisms and anomalously low values for the extremal index, which suggests dominant impacts of positive feedbacks. The observed link between proximity to tipping behaviour and change in the dynamics detected via EVT confirms previous findings obtained by looking at energy fluctuations if fluid flows⁴². This is most evident for warmer climates with respect to colder ones (see Figs. 2, 3), although the latter also show larger than average values of *d* and lower than average values of θ . The instability in the extent of ice sheets during the Icehouse states is also associated with the anomalously high *d* values found in this period. Our finding of an increased persistence (stability) of the Icehouse climate state can be physically linked with a relatively stable Antarctic ice cover over extended periods despite varying short-term climate conditions.

Furthermore, we find that positive feedbacks are more effective on specific orbital periods under different climate states. We detect that they acted on a dominant 23-ky (precession) timescale during Hothouse and Warmhouse climate states, mixed 41-ky (obliquity) and 100-ky (eccentricity) timescales during the Icehouse state, and that there is a remarkable lack of dominant timescales during the Coolhouse state. The detection of the primary variability timescales provides fundamental insights into the drivers of long-term climate variability and confirms the multiscale nature of climate variability^{43,44}. This is valuable information for understanding their significance in the context of future climate changes, and for assessing the capability of (paleo-) climate models to adequately replicate climate states and critical transitions between and within them. Crucially, our results suggest that there may be no simple and unique conceptual model of interconnected feedback for reproducing paleoclimate variability associated with the metastable states above are characterized by different numbers of feedback mechanisms that operate over different timescales for each climate state, and that this has considerable impacts on each climate state's inherent (in-)stability.

Our study establishes a quantitative framework for assessing climate stability and critical transitions across multiple timescales, addressing the limitations of previous classifications that overlook intra-state variability^{3,18}. By applying Multivariate Empirical Mode Decomposition (MEMD) to paleoclimate time series, we uncover previously unrecognized fluctuations within established regimes such as the Hothouse and Icehouse. Extreme Value Theory (EVT) metrics, including instantaneous dimension and extremal index, further quantify stability variations, moving beyond qualitative assessments. These results refine our understanding of climate sensitivity by revealing how feedback mechanisms—particularly volcanic activity, greenhouse gas fluctuations, and carbon cycle dynamics—drive stability shifts on sub-precessional to orbital timescales. Additionally, by linking dynamical systems concepts like local dimension to conventional climate diagnostics, we enhance their applicability in paleoclimate research. Ultimately, our framework not only deepens insight into feedback-driven climate stability but also offers a tool to improve climate modeling by explicitly capturing the dynamic nature of stability across scales.

Methods

Empirical mode decomposition (EMD)

The Empirical Mode Decomposition (EMD) conforms with the class of adaptive decomposition methods and it allows us to decompose a time series s(t) (e.g., δ^{13} C or δ^{18} O) into a finite number n_k of oscillating patterns $c_k(t)$, known as Intrinsic Mode Functions (IMFs), and a monotonic residue r(t) as

$$s(t) = \sum_{k=1}^{n_k} c_k(t) + r(t).$$
(1)

The extraction of Intrinsic Mode Functions (IMFs) from a given signal s(t) is accomplished through the iterative sifting process²⁵. This process systematically isolates oscillatory components in the signal by enforcing certain criteria on the extracted functions. The steps involved are as follows.

- 1. The first step involves detecting all local extrema in s(t), i.e., local maxima and local minima.
- 2. The identified local maxima are interpolated using a cubic spline function to generate the upper envelope u(t), while the local minima are similarly interpolated to derive the lower envelope $\ell(t)$.
- 3. The mean envelope m(t) is then computed as the average of the upper and lower envelopes:

$$m(t) = \frac{u(t) + \ell(t)}{2}.$$
 (2)

4. The detail component h(t), which represents the deviation of the signal from the mean envelope, is obtained by subtracting m(t) from s(t):

$$h(t) = s(t) - m(t).$$
 (3)

- 5. To determine whether h(t) is an IMF, it must satisfy two mathematical conditions:
 - The number of local extrema and the number of zero crossings must either be equal or differ by at most one; and
 - The mean envelope m(t) must be approximately zero.

- 6. If h(t) does not satisfy the above conditions, it is treated as a new signal, and the sifting process (steps 1–5) is repeated on h(t) instead of s(t). This iteration continues until the extracted function meets the IMF criteria. The resulting function is then designated as the first IMF, usually denoted as $c_1(t)$.
- 7. After obtaining $c_1(t)$, it is subtracted from the original signal to obtain the residual component:

$$r_1(t) = s(t) - c_1(t).$$
(4)

The residual $r_1(t)$ serves as the new input signal for the next iteration of the EMD process, where steps 1–6 are repeated to extract the second IMF $c_2(t)$. This process continues iteratively, extracting successive IMFs $c_k(t)$, until the residual $r_k(t)$ becomes a monotonic function or a non-oscillatory trend that cannot be decomposed further.

The decomposition halts when no more oscillatory components (IMFs) can be extracted, leaving only a monotonic residue r(t). This iterative sifting process ensures that each IMF represents a meaningful intrinsic oscillation in the signal, capturing local time-scale variations without relying on predefined basis functions.

The EMD provides a representation of the system as a sum of fluctuating contributions at different average timescales²⁵, although each of them is a non-stationary function with a time-dependent amplitude and phase, i.e., $c_k(t) = a_k(t) \cos [\phi_k(t)]$. The instantaneous amplitude $a_k(t)$ and phase $\phi_k(t)$ are derived via the Hilbert transform. The reader is referred to²⁵ for further detail.

Uni-variate metrics

Instantaneous dimension

In a system described by the time-evolution of a given variable, i.e., via a time series s(t), each time instant can be seen as a *state* of the system that can eventually be visited several times in the future, and whose dynamical properties can be investigated by combining recurrence and extreme value theory⁴⁵. For any given state of interest ζ , the logarithmic return of each state except ζ is

$$g(t) = -\log\left[\delta(s(t),\zeta)\right] \tag{5}$$

where δ is the Euclidean distance between two state vectors. As s(t) approaches ζ then g(t) goes to infinity. If we define a threshold s(q) as the *q*-th empirical quantile of g(t), we can introduce the exceedances $u(\zeta) \doteq \{t \mid g(t) > s(q)\}$, which represent the occurrences that exceed the neighborhood of the reference state. This concept was first introduced by Poincaré⁴⁶ and is akin to the peaks-over-threshold approach widely used in extreme value theory. According to the Freitas-Freitas-Todd theorem⁴⁷, the cumulative probability distribution $F(u, \zeta)$ follows the exponential form of the Generalized Pareto Distribution (GPD):

$$F(u,\zeta) \simeq \exp\left[-\frac{u(\zeta)}{\varsigma(\zeta)}\right].$$
 (6)

In simpler terms, this means that the likelihood of extreme values (large *u*) decreases exponentially, with the rate of decay controlled by the parameter ς . This parameter determines the "thickness" of the tail of the distribution—larger values of ς correspond to heavier tails, meaning extreme events are more frequent. Moreover, ς depends on the system's current state, ζ , and can be used to define an instantaneous dimension, given by $d(\zeta) = \varsigma(\zeta)^{-1}$. This dimension acts as an indicator of how many active feedback mechanisms influence the system at a given

state in phase space^{26,27,45,48}. In essence, a lower instantaneous dimension (higher ς) suggests a system dominated by fewer but stronger interactions, whereas a higher dimension (lower ς) implies more contributing factors regulating the dynamics. However, it is essential to note that from a practical perspective, this instantaneous dimension needs to be understood in a relative sense⁴⁹. Indeed, averaging over the phase-space gives an estimation of the information dimension of the system, i.e., how much space the system explores and how intricate its patterns are, while the gradient (increase or decrease) of the instantaneous dimension indicates structures which feature more or less feedback mechanisms.

Although it seems to be an abstract rather a practical concept, the instantaneous dimension d allows us to infer possible physical processes operating on a system. It illustrates the system's sensitivity to external perturbations. A higher instantaneous dimension indicates a greater number of active modes or mechanisms that can respond to changes, suggesting increased complexity and potential for variability. In the context of climate dynamics, this can be linked to phenomena such as sudden climate shifts or the emergence of climate patterns, where multiple interacting processes come into play. This is because the instantaneous dimension is inherently linked to the physics governing the system. In simple terms, a state with a low value of d is more likely to change in a similar way to its neighboring states as compared to a state with a high d.

The instantaneous dimension provides a valuable lens through which to interpret the complexity of paleoclimatic systems. Specifically, Ref.⁵⁰ demonstrated how, alongside other dynamical systems metrics, could reveal distinct differences in the dynamics of Holocene climates under varying boundary conditions. For instance, their analysis of three simulations—one with standard boundary conditions, one with added vegetation over the Sahara, and one with both vegetation and reduced dust—showed marked variations in the persistence of precipitation patterns and the coupling with sea-level pressure. Higher values of *d* in certain configurations reflected a broader range of active modes in the system, corresponding to increased variability, while lower values indicated more stable and predictable climate patterns. These metrics therefore provide an efficient way to distinguish complex shifts in paleoclimatic simulations, thus offering a complementary perspective to traditional climatological analyses by quantifying dynamical properties linked to potential climate feedbacks and shifts⁵⁰.

Extremal index

The extremal index (θ) is a key concept in extreme value theory, particularly when analyzing clusters of extreme events. In the context of the GPD, it accounts for whether extreme values are isolated or tend to appear in bursts over time and it corrects for this clustering effect when estimating the probability of extreme events. It is evaluated by using the Süveges maximum likelihood estimator^{51,52}, which provides information on the time spent by the system in a given state ζ :

$$\theta = \frac{\sum_{i=1}^{N} \rho S_i + N - 1 + N_c - 2\sum_{i=1}^{N-1} \rho S_i}{2\sum_{i=1}^{N-1} \rho S_i + N - 1 + N_c - 2\sum_{i=1}^{N-1} \rho S_i}$$
(7)
$$\frac{-\left[\left(\sum_{i=1}^{N-1} \rho S_i + N - 1 + N_c\right)^2 - 8N_c \sum_{i=1}^{N-1} \rho S_i\right]^{1/2}}{2\sum_{i=1}^{N-1} \rho S_i}.$$

Here, N represents the number of observations exceeding a defined threshold, ρ is the distribution function of the selected threshold, S_i denotes the exceeding distance, and $N_c = \sum_{i=1}^{N-1} I(S_i \neq 0)$, where I is the indicator

function for the selected S_i . Further details on the calculation can be found in⁵¹.

The extremal index $\theta(\zeta) \in [0, 1]$: when $\theta = 1$, extreme events are independent, meaning that when an extreme value occurs, it is unlikely to be followed by another one; $\theta < 1$, extreme values tend to cluster, meaning that once an extreme event happens, there is a higher chance that another one will follow closely; $\theta \to 0$, extreme values are highly dependent, forming long clusters of extreme events^{27,52}. In straightforward terms, θ represents the inverse of the average persistence time of trajectories near a specific point, and thus informs us for how long the system will reside in a specific region of the phase-space, i.e., in a specific state. An exploration of all states (i.e., all time instants) provides an instantaneous view of the persistence of the system into different states. Consequently, each state ζ of the system, corresponding to the time instant *t* of the time series, is now described by the pair (d, θ) , which are both closely related to the predictability associated with a particular system state. These metrics have offered fresh insights and a different perspective on various geophysical extreme phenomena⁵³⁻⁶⁰.

Bi-variate metrics

The two metrics presented before enable us to retain information about a given system within a *uni-variate* framework, i.e., as described via a single variable s(t). We can extend this formalism to the *bi-variate* case by considering a system described by a pair of variables, i.e., x(t) and y(t). If we define their associated reference state as $\zeta = \{\zeta_x, \zeta_y\}$, the joint logarithmic return is

$$g(x,y;\zeta) = -\frac{1}{2}\log\left[\delta(x(t),\zeta_x)^2 + \delta(y(t),\zeta_y)^2\right].$$
(8)

As for the uni-variate case we can compute the *co-dimension* d_{xy} , representing the mutual number of feedback given by x and y in terms of their joint recurrences, or in other words implying that a given reference state ζ is simultaneously observed in both variables. similarly, the bi-variate extremal index θ_{xy} can be defined as a weighted average of θ_x and θ_y^{61} .

In the bi-variate framework an additional dynamical system metric can be introduced that provides a measure of the mutual coupling between x and y. It is known as the co-recurrence ratio α

$$\alpha = \frac{\# [g(\mathbf{x}; \zeta_x) > g_q(\mathbf{x}; \zeta_x) | g(\mathbf{y}; \zeta_y) > g_q(\mathbf{y}; \zeta_y)]}{\# [g(\mathbf{x}; \zeta_x) > g_q(\mathbf{x}; \zeta_x)]},$$
(9)

where $\#[\cdot]$ denotes the number of events satisfying the condition $[\cdot]$. It measures the percentage of states ζ for which *x* resembles ζ_x , given that *y* resembles ζ_y . If $\alpha = 0$, there are no mutual co-recurrences; if $\alpha = 1$ then a stronger coupling is present between *x* and *y*. However, due to the Bayesian formulation of Eq. (9) α cannot be interpreted in terms of causation but only as a measure of mutual relation between the variables⁶¹.

Instantaneous scale-dependent metrics

The previous discussion introduced the concepts of instantaneous dimension d and inverse persistence θ to provide a local view of phase-space trajectory properties. This allows us to obtain information for each sampled point contributing to the global structure of the phase-space under study. However, in the case of multi-scale systems characterized by processes occurring over a wide range of scales, a scale-dependent phase-space structure can emerge⁶². To obtain a scale-dependent instantaneous view of such systems, a combination of the Empirical Mode Decomposition (EMD) method and extreme value theory is used.

For a multi-scale system described by s(t), we can express it as:

$$s(t) = \langle s(t) \rangle + \sum_{\tau} \delta s^{(\tau)}(t), \qquad (10)$$

where $\langle s(t) \rangle$ represents a steady-state time-averaged value, and $\delta s^{(\tau)}(t)$ is a component of the system operating at a mean scale τ . An analogy can be drawn between Eqs. (10) and (1) with the correspondence $c_k(t) \leftrightarrow \delta s^{(\tau)}(t)$

and $r(t) \leftrightarrow \langle s(t) \rangle$. This means that for each scale τ , we can identify the corresponding invariant set $M\tau$ as the manifold obtained via the partial sums of Intrinsic Mode Functions (IMFs) with scales $\tau \star < \tau$:

$$s^{\tau}(t) = \sum_{k^{\star}=1}^{k} c_{k^{\star}}(t), \tag{11}$$

For each scale $\tau \in [\tau_1, \tau_{n_k}]$, where n_k is the number of IMFs, given a trajectory $s^{\tau}(t)$ and a state of interest ζ_{τ} , the cumulative probability of logarithmic returns in the neighborhood of ζ_{τ} follows a GPD:

$$F(u_{\tau},\zeta_{\tau}) \simeq \exp\left[-\theta_{\tau}(\zeta_{\tau}), \frac{u_{\tau}(\zeta_{\tau})}{\varsigma_{\tau}(\zeta_{\tau})}\right].$$
(12)

Thus, two scale-dependent metrics $d(t, \tau) = \varsigma_{\tau}(\zeta_{\tau})^{-1}$ and $\theta(t, \tau)$ can be introduced, representing the number of active feedback and the stability/persistence of fluctuations up to a maximum scale of τ around each state ζ_{τ} . In a similar fashion we can introduce, in a bi-variate framework, the scale-dependent co-recurrent ratio $\alpha(t, \tau)$. By using the EMD to derive scale-dependent components within the system and extreme value theory-based metrics to obtain the instantaneous scale-dependent metrics, this approach provides valuable insights into the system's behavior at different scales^{23,24}.

Data availibility

All data are available open access in electronic form at the PANGAEA data repository (https://doi.org/10.1594 /PANGAEA.917503).

Code availability

The code to perform the analogues dynamical analysis is available at https://fr.mathworks.com/matlabcentral/fi leexchange/95768-attractor-local-dimension-and-local-persistence-computation.

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Author contributions

TA and FF designed research, TA, EJR, and VL performed research, TA and DF analyzed data. All authors contributed to discussing and writing the paper.

Declarations

Competing interests

The authors declare no competing interests nor conflicts of interest. No human or animal data have been used in this study.

Additional information

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