SUPPLEMENTARY INFORMATION

Sea-level and deep-sea temperature variability of the past 5.3 million years

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APPENDIX

PTC MathCad worksheet for calculation of the RSL_{Gib} relationship with eastern Mediterranean $\delta^{18}O$ as measured on the planktonic foraminiferal species *Globigerinoides ruber* (white) and *Neogloboquadrina pachyderma* (dextral).

Note that, in the list below, we mention “subscript”, but that subscripts are not offset downward in MathCad notation.
The final work reported in our study used N=500 (N is the number of iterations for the matrix from which the probability intervals are calculated).

Ap = volume of present-day Atlantic inflow through Strait of Gibraltar
Area = area of Mediterranean Sea
B = net freshwater input from Black Sea
E = evaporation
ea = saturation vapour pressure at temperature of air 10 m above sea-level
es = saturation vapour pressure at sea surface temperature
L = latent heat of vaporisation
P = precipitation
p = total pressure at mean sea level in Pa
qa = saturation mixing ratio at temperature of air 10 m above sea-level
qs = saturation mixing ratio at sea surface temperature
Qin = Atlantic inflow volume (through St. of Gibraltar)
r = relative humidity
R = runoff
res = results matrix
Sinp = present-day inflow salinity (in St. of Gibraltar)
SL = sea level
sl = –SL
subscript “atm” = indicator for value in overlying atmosphere
subscript “c” = indicator for condensation temperature (cloud base)
subscript “in” = indicator of inflow value
subscript “init” = indicator of initial value, changed later in the calculation-loop
subscript “p” = present-day
subscript “s” = summer
subscript “sap” = indicator of past value
subscript “sml” = summer mixed layer
subscript “ssth” = summer sub-thermocline layer (winter water below summer thermocline)
subscript “w” = winter

subscript “wml” = winter mixed layer
t = residence time
T = water temperature
Ta = air temperature
V = wind speed
Vol = volume
X = excess of evaporation over all freshwater input
z = depth
α = factor for pycnocline shoaling
γ = factor for buoyancy loss change
δ = $\delta^{18}O$ (oxygen isotope ratio)
δB = mean oxygen isotope ratio of B
δinp= present-day inflow oxygen isotope ratio of water (in St. of Gibraltar)
δR = mean oxygen isotope ratio of runoff
ρ = density of air at mean sea-level pressure of 1012 mbar
Ω = factor for Qin change due to buoyancy loss change (γ)
x = fraction of summer mixed-layer depth relative to winter mixed-layer depth
αs = water oxygen isotope fractionation factor at evaporation (summer)
αw = water oxygen isotope fractionation factor at evaporation (winter)
αcs = water oxygen isotope fractionation factor at condensation (summer)
αcw = water oxygen isotope fractionation factor at condensation (winter)
αcalc = fractionation factor for oxygen isotopes at calcification

augment = operator for collating columns
ceil = operator for rounding up to nearest integer
rnorm = operator for random number selection from normal distribution
stack = operator for collating rows
submatrix = operator for reading out part of a matrix

References for the worksheet

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modern values used

wind speed:	$V := 7.5$	m s^{-1}	Average wind speed 1945-1990 (Garrett et al., 1993)

Annual average Temp = 19C (Stanev et al., 1989; Nykjaer, 2009)
Summer half year 22 C. Winter halfyear 16 C

precipitation volume: $P := 1.25 \cdot 10^{12}$ m^3, or ~0.4*E (Garrett et al., 1993)

Set percentage of the (random) uncertainty of Black Sea net Freshwater flux for summer. Note, this is a negligible term in the Black Sea fluxes considered in the present study, and is in the model for other work $sinc := 25$ %

Common numbers:

Exchange coefficient:	$C := 1.15 \cdot 10^{-3}$		(Garrett et al., 1993; Wells, 1995)
Density of air:	$\rho := 1.2$	kg m^{-3}	(at mean sea level pressure = 1012 mbar)
Area of basin:	$Area := 2.5 \cdot 10^{12}$	m^2	
Total pressure	$p := 1.012 \cdot 10^{5}$	Pa	at mean sea level

Modern values:

$Sinp \equiv 36.2$
$\delta inp \equiv 1$

NB. The three-lined operator means the same as =, but does not have to be used in strict succession in the worksheet to function.

$Xp := 1.3 \cdot 10^{12}$

$Ap := 23.0 \cdot 10^{12}$

Bryden et al. (1994)

$Rp := 0.45 \cdot 10^{12}$ m^3 (roughly 0.2 m yr^{-1}; Garrett et al., 1993)

$Bp := 0.2 \cdot 10^{12}$ m^3 (before damming; Tolmazin, 1985)

$\delta B := -9$ $\delta R := -6$

Set number of iterations for probabilisitc assessment:

$N \equiv 500$

Notes:

For carbonate-based oxygen isotopes isotopes (denoted with "c", fractionation relative to water-based isotope values are calculated after NIST (1992) and Coplen et al. (1983), using box temperatures and a 1.0309 calibration factor between SMOW and PDB

Subdivision of thicknesses of summer mixed layer and summer subthermocline layer, which added together make the winter mixed layer (modern winter mixed layer depth = ~150 or more m, and summer mixed layer depth = ~30± 5m (Nykjaer 2009)

Reminder: in the model here, sap stands for "past", not for sapropel time

Bethoux and Gentilli (1994) say that E =~ 100cm/y and Gilman and Garrett (1994) say that net freshwater flux is 71±7 cm/y (see Artale et al., 2002). Bryden et al (1994) and Garrett et al (1993) argue that Xp/Area is 0.52 to 0.66 m/y. For realistic S and δ gradients, the model here diagnoses ~60cm/y

Matrix initialisations. Will be removed later from output matrix out, but remains as first line in "Results". So the first row with all zeros will need to be manually removed from the final output file.

$res0 := (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$

$out0 := (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$

Results :=

"make an initial (token) start for the results output matrix"

$res \leftarrow res0$

for $f \in 0..36$

"set a sea-level range for solutions"

$SL \leftarrow 30 - f \cdot 5$

"make an initial (token) start for the loop output matrix (later removed) "

$out \leftarrow out0$

for $a \in 1..N$

"set a sign convention for sea-level lowering (model uses lowering as a + value)"

$sl \leftarrow -SL$

"set relative humidity, after Rohling (1999), and so that modern freshwater balance and salinities agree with observations"

$$r \leftarrow 0.7 + \frac{0.05}{3} \cdot rnorm(1,0,1)$$

"set wind speed to modern annual mean (Garrett et al., 1993), with shift to stronger (winter-type)values at glacial times, and with a 3 sigma uncertainty of 1 m/s"

$$V \leftarrow 7.5 + 1 \cdot \frac{sl}{120} + \frac{1}{3} \cdot rnorm(1,0,1)$$

"set depth of summer mixed layer (zsml) to 30±5 m where 5 is a 3sigma range (Nykjaer, 2009)"

$$zsml \leftarrow 30 + \frac{5}{3} \cdot rnorm(1,0,1)$$

"set isotopic values for dR and dB to modern values with 3 sigma ranges over ±1 ppt"

$$\delta Rw \leftarrow \delta R - 1 + \frac{1}{3} \cdot rnorm(1,0,1)$$

$$\delta Rs \leftarrow \delta R + 1 + \frac{1}{3} \cdot rnorm(1,0,1)$$

$\delta Pw \leftarrow \delta Rw$

$\delta Ps \leftarrow \delta Rs$

$$\delta Bw \leftarrow \delta B + \frac{1}{3} \cdot rnorm(1,0,1)$$

$$\delta Bs \leftarrow \delta B + \frac{1}{3} \cdot rnorm(1,0,1)$$

"set R and B fluxes to modern values with uncertainties over 3 sigma ranges of ± 10 %"

"B is set to zero when sea level is below 80 or more m below present"

$$B \leftarrow \begin{cases} \left(1 + \frac{0.1}{3} \cdot rnorm(1,0,1)\right) \cdot Bp & \text{if } sl \le 80 \\ 0 & \text{if } sl > 80 \end{cases}$$

"set T changes after Hayes et al (2005). Summer 23 to 19 = 4 deg. C and Winter 16 to 12.5 = 3.5 deg. C"

"apply those glacial-interglacial gradietns relative to modern Ts = 22 and Tw =16 deg. C, after Nykjaer (2009) and Stanev (see Rohling, 1999)"

$$Ts \leftarrow 22 - \left(5 + \frac{1}{3} \cdot rnorm(1,0,1)\right) \cdot \frac{sl}{120}$$

$$Tw \leftarrow 16 - \left(3.5 + \frac{1}{3} \cdot rnorm(1,0,1)\right) \cdot \frac{sl}{120}$$

"set summer and winter air-sea T differences as described in note to the side"

$$\Delta Ts \leftarrow 0.5 + \left(1 + \frac{1}{3} \cdot rnorm(1,0,1)\right) \cdot \frac{sl}{120}$$

"....."

Perform calculations N times, which gives the matrix of results over which probability intervals are determined

$\Delta Tw \leftarrow 1.5 + \frac{1}{3} \cdot rnorm(1,0,1)$

$Tssth \leftarrow Tw$

$Tint \leftarrow Tw - \left(1 + \frac{1}{3} \cdot rnorm(1,0,1)\right)$

"set Atlantic inflow T so that there is no net heat gain/loss in Mediterranean beyond a 3 sigma = ±1 deg. C difference"

$Tatlantic \leftarrow Tint + \frac{1}{3} \cdot rnorm(1,0,1)$

$Tas \leftarrow Ts - \Delta Ts$

$Taw \leftarrow Tw - \Delta Tw$

"calculate SUMMER half-year evaporation"

"Following Abbott and Tabony (1985)"

$L \leftarrow (2500.84 - 2.34 \cdot Ts) \cdot 10^3$

$ess \leftarrow 10^2 \cdot e^{55.17 - 6803 \cdot (Ts + 273.15)^{-1} - 5.07 \cdot \ln(Ts + 273.15)}$

$eas \leftarrow 10^2 \cdot e^{55.17 - 6803 \cdot (Tas + 273.15)^{-1} - 5.07 \cdot \ln(Tas + 273.15)}$

"Then, following Wells (1986)"

$qss \leftarrow \frac{ess}{p - ess} \cdot \left(\frac{18.0153}{28.965}\right)$

$qas \leftarrow \frac{eas}{p - eas} \cdot \left(\frac{18.0153}{28.965}\right)$

"calculate Es in m3/y over the summer HALF year"

$Es \leftarrow \frac{\rho \cdot L \cdot C \cdot V \cdot (qss - r \cdot qas) \cdot Area \cdot 1.26 \cdot 10^{-2}}{2}$

"maintain modern E:P proportion (Garrett et al., 1993)"

$Ps \leftarrow 0.4 \cdot Es$

"change amount of runoff in proportion to change in amount of precipitation"

$Rs \leftarrow \left(1 + \frac{0.1}{3} \cdot rnorm(1,0,1)\right) \cdot Rp \cdot \frac{Ps}{P}$

"calculating WINTER half-year evaporation"

"Following Abbott and Tabony (1985)"

$L \leftarrow (2500.84 - 2.34 \cdot Tw) \cdot 10^3$

$esw \leftarrow 10^2 \cdot e^{55.17 - 6803 \cdot (Tw + 273.15)^{-1} - 5.07 \cdot \ln(Tw + 273.15)}$

$eaw \leftarrow 10^2 \cdot e^{55.17 - 6803 \cdot (Taw + 273.15)^{-1} - 5.07 \cdot \ln(Taw + 273.15)}$

"Then, following Wells (1986; p.83)"

$qsw \leftarrow \frac{esw}{p - esw} \cdot \left(\frac{18.0153}{28.965}\right)$

$qaw \leftarrow \frac{eaw}{p - eaw} \cdot \left(\frac{18.0153}{28.965}\right)$

"calculating Ew in m3/y over the summer HALF year"

$Ew \leftarrow \frac{\rho \cdot L \cdot C \cdot V \cdot (qsw - r \cdot qaw) \cdot Area \cdot 1.26 \cdot 10^{-2}}{2}$

" "

Regarding ΔT, Artale et al (2002 JGR) argue that it should be very small in strong wind conditions. We use different ΔT values for summer and winter for the Present, as summarised in Rohling (1999), but we let ΔTs become equal to ΔTw at full glacial times (= 'locking' the basin in modern winter mode)

.....

$Pw \leftarrow 0.4 \cdot Ew$

"change amount of runoff in proportion to change in amount of precipitation"

$Rw \leftarrow \left(1 + \frac{0.1}{3} \cdot rnorm(1,0,1)\right) \cdot Rp \cdot \frac{Pw}{P}$

"calculate freshwater budget"

$Bs \leftarrow \frac{Bp}{2} + \frac{sinc}{100} \cdot (B - Bp)$

$Bw \leftarrow \frac{Bp}{2} + \frac{100 - sinc}{100} \cdot (B - Bp)$

$Xs \leftarrow Es - Ps - Rs - Bs$

$Xw \leftarrow Ew - Pw - Rw - Bw$

"set past inflow salinity and oxygen isotope value as function of present inflow values plus general glacial enrichment"

$Sinsap \leftarrow Sinp + \frac{sl}{120} \cdot 1$

$\delta insap \leftarrow \delta inp + sl \cdot 0.009$

"set exchange reduction factor PHI as function of sea level, after Rohling (1991a, 1994, 1999; Rohling and Bryden, 1994)"

$\Phi \leftarrow 1 - 0.5 \cdot \frac{sl}{120}$

"set exchange reduction factor gamma as function of buoyancy loss, after Rohling (1991b, 1994, 1999; Rohling and Bryden, 1994)"

"then determine exchange relative to the present"

$Xsap \leftarrow Xs + Xw$

$\gamma \leftarrow \frac{Xsap}{Xp}$

$\Omega \leftarrow \gamma^{\frac{1}{3}}$

$Qinp \leftarrow Ap$

$Qinsap \leftarrow \Omega \cdot \Phi \cdot Qinp$

$Qoutsap \leftarrow Qinsap - Xsap$

"subdivide inflow over sml and ssth; first calculate pycnocline shoaling, after Rohling & Bryden (1994)"

$\alpha \leftarrow \frac{\frac{Ap}{Xp} \cdot \Phi \cdot \Omega - \gamma}{\frac{Ap}{Xp} - 1} \cdot \frac{Sinp}{Sinsap}$

"set modern depth of winter mixed layer (zwml) at 150 m (minimum value in Nykjaer, 2009)"

"zwml - zsml then gives thickness of summer sub-thermocline layer (zssth)"

$zssth \leftarrow (\alpha \cdot 150) - zsml$

"calculate the fractionation factors and residence times"

$Tcw \leftarrow Tw - \left(5 + \frac{1}{3} \cdot rnorm(1,0,1)\right)$

$Tcs \leftarrow Ts - \left(5 + \frac{1}{3} \cdot rnorm(1,0,1)\right)$

$\alpha s \leftarrow e^{\frac{1.137}{(Ts+273.15)^2} \cdot 10^3 - \frac{0.4156}{Ts+273.15} - 2.0667 \cdot 10^{-3}}$

$\alpha w \leftarrow e^{\frac{1.137}{(Tw+273.15)^2} \cdot 10^3 - \frac{0.4156}{Tw+273.15} - 2.0667 \cdot 10^{-3}}$

"....."

$\delta Eeqs \leftarrow -\ln(\alpha s)\cdot 10^{3}$

$\delta Eeqw \leftarrow -\ln(\alpha w)\cdot 10^{3}$

$\alpha cs \leftarrow e^{\frac{1.137}{(Tcs+273.15)^2}\cdot 10^{3}-\frac{0.4156}{Tcs+273.15}-2.0667\cdot 10^{-3}}$

$\alpha cw \leftarrow e^{\frac{1.137}{(Tcw+273.15)^2}\cdot 10^{3}-\frac{0.4156}{Tcw+273.15}-2.0667\cdot 10^{-3}}$

$\delta atms \leftarrow \delta Rs - \ln(\alpha cs)\cdot 10^{3}$

$\delta atmw \leftarrow \delta Rw - \ln(\alpha cw)\cdot 10^{3}$

$Zs \leftarrow \frac{qas}{qss}$

$Zw \leftarrow \frac{qaw}{qsw}$

$x \leftarrow \frac{zsml}{zssth + zsml}$

$Volssth \leftarrow Area\cdot zssth$

$Volsml \leftarrow Area\cdot zsml$

$tsml \leftarrow \frac{Volsml}{x\cdot Qinsap}$

$T \leftarrow ceil(tsml)$

$\alpha calcTs \leftarrow e^{\frac{2.78}{(Ts+273.15)^2}\cdot 10^{3}-3.39\cdot 10^{-3}}$

$\alpha calcTw \leftarrow e^{\frac{2.78}{(Tw+273.15)^2}\cdot 10^{3}-3.39\cdot 10^{-3}}$

$\alpha calcTint \leftarrow e^{\frac{2.78}{(Tint+273.15)^2}\cdot 10^{3}-3.39\cdot 10^{-3}}$

$\alpha calcTatlantic \leftarrow e^{\frac{2.78}{(Tatlantic+273.15)^2}\cdot 10^{3}-3.39\cdot 10^{-3}}$

$\delta init \leftarrow \delta insap$

$Sinit \leftarrow Sinsap$

"now do the box-loop calculations"

for $k \in 0..500$

$\delta Es \leftarrow \delta init + \delta Eeqs$

$Ssml \leftarrow \frac{\left(\frac{1}{T}\cdot Sinsap + \frac{T-1}{T}\cdot Sinit\right)\cdot Volsml}{Volsml - Xs}$

$\delta sml \leftarrow$

$\delta sml \leftarrow \frac{\left(\frac{1}{T}\cdot \delta insap + \frac{T-1}{T}\cdot \delta init\right)\cdot Volsml + Rs\cdot \delta Rs + Bs\cdot \delta Bs + Ps\cdot \delta Ps - Es\cdot \delta Es}{Volsml - Xs}$

for $m \in 1..50$

$\delta Es \leftarrow \left[\frac{\frac{1+\delta sml\cdot 10^{-3}}{\alpha s} - Zs\cdot r - \delta atms\cdot 10^{-3}\cdot Zs\cdot r}{1.0142\cdot(1 - Zs\cdot r)} - 1\right]\cdot 10^{3}$

"....."

Loop set to iterate k = 500 times to ensure steady state is achieved in each box once fluxes and residence times are adjusted

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$$\delta sml \leftarrow \frac{\left(\frac{1}{T}\cdot \delta insap + \frac{T-1}{T}\cdot \delta init\right)\cdot Volsml + Rs\cdot \delta Rs + Bs\cdot \delta Bs + Ps\cdot \delta Ps - Es\cdot \delta Es}{Volsml - Xs}$$

δsml

$$Sssth \leftarrow \frac{\left(\frac{1}{T}\cdot Sinsap + \frac{T-1}{T}\cdot Sinit\right)\cdot Volssth}{Volssth}$$

$$\delta ssth \leftarrow \frac{\left(\frac{1}{T}\cdot \delta insap + \frac{T-1}{T}\cdot \delta init\right)\cdot Volssth}{Volssth}$$

$Qsml \leftarrow Volsml - Xs$

$Qssth \leftarrow Volssth$

$$\delta Ew \leftarrow \frac{zsml\cdot \delta sml + zssth\cdot \delta ssth}{zsml + zssth} + \delta Eeqw$$

$$Swml \leftarrow \frac{Qsml\cdot Ssml + Qssth\cdot Sssth}{Qsml + Qssth - Xw}$$

$$\delta wml \leftarrow \left| \delta wml \leftarrow \frac{Qsml\cdot \delta sml + Qssth\cdot \delta ssth + Rw\cdot \delta Rw + Bw\cdot \delta Bw + Pw\cdot \delta Pw - Ew\cdot \delta Ew}{Qsml + Qssth - Xw} \right.$$

for $h \in 1..50$

$$\delta Ew \leftarrow \left[\frac{\frac{1 + \delta wml\cdot 10^{-3}}{\alpha w} - Zw\cdot r - \delta atmw\cdot 10^{-3}\cdot Zw\cdot r}{1.0142\cdot (1 - Zw\cdot r)} - 1\right]\cdot 10^{3}$$

$$\delta wml \leftarrow \frac{Qsml\cdot \delta sml + Qssth\cdot \delta ssth + Rw\cdot \delta Rw + Bw\cdot \delta Bw + Pw\cdot \delta Pw - Ew\cdot \delta Ew}{Qsml + Qssth - Xw}$$

δwml

$Sinit \leftarrow Swml$

$\delta init \leftarrow \delta wml$

$$\delta smlc \leftarrow 10^{3}\cdot \ln(\alpha calcTs) + \frac{\delta sml - 30.92}{1.03092}$$

$$\delta ssthc \leftarrow 10^{3}\cdot \ln(\alpha calcTw) + \frac{\delta ssth - 30.92}{1.03092}$$

$$\delta wmlc \leftarrow 10^{3}\cdot \ln(\alpha calcTw) + \frac{\delta wml - 30.92}{1.03092}$$

$$\delta intc \leftarrow 10^{3}\cdot \ln(\alpha calcTint) + \frac{\delta wml - 30.92}{1.03092}$$

$$\delta ac \leftarrow 10^{3}\cdot \ln(\alpha calcTatlantic) + \frac{\delta insap - 30.92}{1.03092}$$

$zwml \leftarrow zsml + zssth$

$outa \leftarrow augment(-sl, Ssml, Sssth, Swml, \delta sml, \delta ssth, \delta wml, \delta smlc, \delta ssthc, \delta wmlc, \delta intc, \delta ac, Sinsap, \delta insap, zsml, zssth, Xsap, Ew, Es, zwml)$

$out \leftarrow stack(out, outa)$

$subout \leftarrow submatrix(out, 1, N, 0, 19)$

$Ssmldata \leftarrow augment\left(median\left(subout^{\langle 1\rangle}\right), percentile\left(subout^{\langle 1\rangle}, 0.16\right) - median\left(subout^{\langle 1\rangle}\right), percentile\left(subout^{\langle 1\rangle}, 0.025\right) - median\left(subout^{\langle 1\rangle}\right), percentile\left(subout^{\langle 1\rangle}, 0.84\right) - median\left(subout^{\langle 1\rangle}\right), percentile\left(subout^{\langle 1\rangle}, 0.975\right) - median\left(subout^{\langle 1\rangle}\right)\right)$

$Sssthdata \leftarrow augment\left(median\left(subout^{\langle 2\rangle}\right), percentile\left(subout^{\langle 2\rangle}, 0.16\right) - median\left(subout^{\langle 2\rangle}\right), percentile\left(subout^{\langle 2\rangle}, 0.025\right) - median\left(subout^{\langle 2\rangle}\right), percentile\left(subout^{\langle 2\rangle}, 0.84\right) - median\left(subout^{\langle 2\rangle}\right), percentile\left(subout^{\langle 2\rangle}, 0.975\right) - median\left(subout^{\langle 2\rangle}\right)\right)$

$Swmldata \leftarrow augment\left(median\left(subout^{\langle 3\rangle}\right), percentile\left(subout^{\langle 3\rangle}, 0.16\right) - median\left(subout^{\langle 3\rangle}\right), percentile\left(subout^{\langle 3\rangle}, 0.025\right) - median\left(subout^{\langle 3\rangle}\right), percentile\left(subout^{\langle 3\rangle}, 0.84\right) - median\left(subout^{\langle 3\rangle}\right), percentile\left(subout^{\langle 3\rangle}, 0.975\right) - median\left(subout^{\langle 3\rangle}\right)\right)$

$\delta smldata \leftarrow augment\left(median\left(subout^{\langle 4\rangle}\right), percentile\left(subout^{\langle 4\rangle}, 0.16\right) - median\left(subout^{\langle 4\rangle}\right), percentile\left(subout^{\langle 4\rangle}, 0.025\right) - median\left(subout^{\langle 4\rangle}\right), percentile\left(subout^{\langle 4\rangle}, 0.84\right) - median\left(subout^{\langle 4\rangle}\right), percentile\left(subout^{\langle 4\rangle}, 0.975\right) - median\left(subout^{\langle 4\rangle}\right)\right)$

$\delta ssthdata \leftarrow augment\left(median\left(subout^{\langle 5\rangle}\right), percentile\left(subout^{\langle 5\rangle}, 0.16\right) - median\left(subout^{\langle 5\rangle}\right), percentile\left(subout^{\langle 5\rangle}, 0.025\right) - median\left(subout^{\langle 5\rangle}\right), percentile\left(subout^{\langle 5\rangle}, 0.84\right) - median\left(subout^{\langle 5\rangle}\right), percentile\left(subout^{\langle 5\rangle}, 0.975\right) - median\left(subout^{\langle 5\rangle}\right)\right)$

"....."

$$\delta wmldata \leftarrow augment\left(median\left(subout^{\langle 6\rangle}\right), percentile\left(subout^{\langle 6\rangle}, 0.16\right) - median\left(subout^{\langle 6\rangle}\right), percentile\left(subout^{\langle 6\rangle}, 0.025\right) - median\left(subout^{\langle 6\rangle}\right), percentile\left(subout^{\langle 6\rangle}, 0.84\right) - median\left(subout^{\langle 6\rangle}\right), percentile\left(subout^{\langle 6\rangle}, 0.975\right) - median\left(subout^{\langle 6\rangle}\right)\right)$$

$$\delta smlcdata \leftarrow augment\left(median\left(subout^{\langle 7\rangle}\right), percentile\left(subout^{\langle 7\rangle}, 0.16\right) - median\left(subout^{\langle 7\rangle}\right), percentile\left(subout^{\langle 7\rangle}, 0.025\right) - median\left(subout^{\langle 7\rangle}\right), percentile\left(subout^{\langle 7\rangle}, 0.84\right) - median\left(subout^{\langle 7\rangle}\right), percentile\left(subout^{\langle 7\rangle}, 0.975\right) - median\left(subout^{\langle 7\rangle}\right)\right)$$

$$\delta ssthcdata \leftarrow augment\left(median\left(subout^{\langle 8\rangle}\right), percentile\left(subout^{\langle 8\rangle}, 0.16\right) - median\left(subout^{\langle 8\rangle}\right), percentile\left(subout^{\langle 8\rangle}, 0.025\right) - median\left(subout^{\langle 8\rangle}\right), percentile\left(subout^{\langle 8\rangle}, 0.84\right) - median\left(subout^{\langle 8\rangle}\right), percentile\left(subout^{\langle 8\rangle}, 0.975\right) - median\left(subout^{\langle 8\rangle}\right)\right)$$

$$\delta wmlcdata \leftarrow augment\left(median\left(subout^{\langle 9\rangle}\right), percentile\left(subout^{\langle 9\rangle}, 0.16\right) - median\left(subout^{\langle 9\rangle}\right), percentile\left(subout^{\langle 9\rangle}, 0.025\right) - median\left(subout^{\langle 9\rangle}\right), percentile\left(subout^{\langle 9\rangle}, 0.84\right) - median\left(subout^{\langle 9\rangle}\right), percentile\left(subout^{\langle 9\rangle}, 0.975\right) - median\left(subout^{\langle 9\rangle}\right)\right)$$

$$\delta intcdata \leftarrow augment\left(median\left(subout^{\langle 10\rangle}\right), percentile\left(subout^{\langle 10\rangle}, 0.16\right) - median\left(subout^{\langle 10\rangle}\right), percentile\left(subout^{\langle 10\rangle}, 0.025\right) - median\left(subout^{\langle 10\rangle}\right), percentile\left(subout^{\langle 10\rangle}, 0.84\right) - median\left(subout^{\langle 10\rangle}\right), percentile\left(subout^{\langle 10\rangle}, 0.975\right) - median\left(subout^{\langle 10\rangle}\right)\right)$$

$$\delta acdata \leftarrow augment\left(median\left(subout^{\langle 11\rangle}\right), percentile\left(subout^{\langle 11\rangle}, 0.16\right) - median\left(subout^{\langle 11\rangle}\right), percentile\left(subout^{\langle 11\rangle}, 0.025\right) - median\left(subout^{\langle 11\rangle}\right), percentile\left(subout^{\langle 11\rangle}, 0.84\right) - median\left(subout^{\langle 11\rangle}\right), percentile\left(subout^{\langle 11\rangle}, 0.975\right) - median\left(subout^{\langle 11\rangle}\right)\right)$$

$$Xsapdata \leftarrow augment\left(\frac{median\left(subout^{\langle 16\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 16\rangle}, 0.16\right) - median\left(subout^{\langle 16\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 16\rangle}, 0.025\right) - median\left(subout^{\langle 16\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 16\rangle}, 0.84\right) - median\left(subout^{\langle 16\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 16\rangle}, 0.975\right) - median\left(subout^{\langle 16\rangle}\right)}{Area}\right)$$

$$Ewdata \leftarrow augment\left(\frac{median\left(subout^{\langle 17\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 17\rangle}, 0.16\right) - median\left(subout^{\langle 17\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 17\rangle}, 0.025\right) - median\left(subout^{\langle 17\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 17\rangle}, 0.84\right) - median\left(subout^{\langle 17\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 17\rangle}, 0.975\right) - median\left(subout^{\langle 17\rangle}\right)}{Area}\right)$$

$$Esdata \leftarrow augment\left(\frac{median\left(subout^{\langle 18\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 18\rangle}, 0.16\right) - median\left(subout^{\langle 18\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 18\rangle}, 0.025\right) - median\left(subout^{\langle 18\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 18\rangle}, 0.84\right) - median\left(subout^{\langle 18\rangle}\right)}{Area}, \frac{percentile\left(subout^{\langle 18\rangle}, 0.975\right) - median\left(subout^{\langle 18\rangle}\right)}{Area}\right)$$

$$zwmldata \leftarrow augment\left(median\left(subout^{\langle 19\rangle}\right), percentile\left(subout^{\langle 19\rangle}, 0.16\right) - median\left(subout^{\langle 19\rangle}\right), percentile\left(subout^{\langle 19\rangle}, 0.025\right) - median\left(subout^{\langle 19\rangle}\right), percentile\left(subout^{\langle 19\rangle}, 0.84\right) - median\left(subout^{\langle 19\rangle}\right), percentile\left(subout^{\langle 19\rangle}, 0.975\right) - median\left(subout^{\langle 19\rangle}\right)\right)$$

$$resa \leftarrow augment(-sl, Ssmldata, Sssthdata, Swmldata, \delta smldata, \delta ssthdata, \delta wmldata, \delta smlcdata, \delta ssthcdata, \delta wmlcdata, \delta intcdata, \delta acdata, Xsapdata, Ewdata, Esdata, zwmldata)$$

$$res \leftarrow stack(res, resa)$$

res